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The Art and Science of Selecting Robot Motors

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This three part series is devoted to answering the question: How do I choose a motor to drive my robot? We will cover all you need to know about selecting a motor to power your robot. There are basically three types of motors that are used on the majority of mobile robots, permanent magnet direct current (PMDC), radio controlled servo (R/C servo), and stepper motors. Since most of hobbyist robots, those that range in size from 8 to 16 inches in overall size, use PMDC motors for locomotion, this series will concentrate on choosing the correct motor for these robot missions. Smaller robots frequently use R/C servo motors, heavy combat robots may use specialized, rare earth magnet motors and outdoor robots may use internal combustion engines. Those will not be covered. My area of interest is primarily in competition robots and consequently the illustrative examples will use robots my students built and entered in contests. Nevertheless, the general principles outlined here will apply to all PMDC motor applications.

This series is divided into three parts: Part I covers deriving motor requirements from the robot's niche – task plus environment; Part II gives the basic equations governing DC motor operations; Part III shows how to apply the lessons learned in Parts I & II with examples from three very different competition robots. There are many PMDC motors to choose from and your choices will depend on a combination of the robot's niche, what's available, what you can afford or adapt, and your experience and preferences The information that follows will show you how to make an informed choice.

Part I. Setting Motor Requirements

Introduction To DC Motors

The good news is that there are many types of motors from which to choose and, as the joke goes, the bad news is that there are many types of motors from which to choose. All motors, even the small DC motors considered here, can be complex and there is a large literature available to give you all the gory details. While we will provide an overview that is sufficient for all your robot needs, I encourage you to search library bookshelves and the Internet for the many excellent tutorials available.

Choosing a motor is a compromise between what we want it to do and what is available at a cost we can afford. The intelligent choice of a motor requires us to understand the workings, advantages, and disadvantages of various motor parameters and to develop a specification for the motor performance characteristics. This will help us to choose the correct motor for the task. We will begin with a brief overview of PMDC motors.

Motors are described by a large number of operating characteristics. We'll list the most important characteristics now and give a fuller explanation of the terms later. The motor characteristics we are most concerned with are:

- Operating Voltage – Various voltages may be given in a motor specification; most commonly, the nominal voltage for continuous operation. Many motors may be operated at more than their rated voltage with increased torque and rotation rate, but may overheat if used for more than a short time. Over volting can be used to advantage in contests with short time limits.
- Motor Speed or Rotation Rate – how fast the motor shaft turns. This angular rate is almost always given as revolutions per minute (RPM), but some times as degrees or radians per second.
- Torque – a measure of a motor's ability to provide a "turning force". When you turn the lid on a jar, you exert a torque, which causes the lid to rotate. In our application, the motor torque is conveyed to a wheel or a lever, which then causes the robot to move or the lever to lift, push, or pull something. Torque is measured in terms of force times the perpendicular distance between the force and the point of rotation, i.e. the lever arm. It is usually given in terms of ounce-inches (oz-inch), gram-centimeters (gm-cm) or foot-pounds (ft-lbs).

- Current Draw – the current in amps (or milliamps). This may be given for different conditions, such as no load (free running), nominal load (with a specific torque), and stall (when the motor shaft doesn't have enough torque to overcome the imposed load and is unable to turn).
- Physical Measurements (in English or Metric units) – separate measurements are usually provided for the overall motor size, the size of the motor shaft, and the mounting plate screw or bolt arrangement.
- Special Features – some motors come with extras, such as an encoder, brake, clutch, right angle gear head, special mounting bracket, or dual output shafts.

PMDC Gear Head Motors

As the name implies, DC motors run off of direct current, the kind of current that is supplied by batteries, which is one of the main reasons that these type of motors are used in robots. Small DC motors vary quite a bit in quality but most have the same essential features. DC motors work by using a basic law of physics which states that a force is exerted on an electrical current passing through a magnetic field. Current traveling through the motor's internal wires, which are surrounded by permanent magnets, generates a force which is communicated to the motor shaft, around which the wires are wound. Reversing the direction, or polarity, of the current changes the rotation direction of the motor shaft from clock-wise (CW) to counter clock-wise (CCW). The speed is altered by varying the voltage (hence current) applied to the motor.

DC motors run at speeds of thousands of RPMs with low torque. This is not suitable for driving a robot. The output torque is much too low to move the robot. In order to use the motor, we add a gearbox, a kind of transmission except that there is no shifting of gears, to reduce the motor speed and increase the output torque. Thus the same motor may produce different torque and speed ratings depending on the gearing used between the motor and the gearbox output shaft. Many DC motors come with a gearbox already attached and these are simply called DC gear head motors and are the type of motors in which we will be most interested. From now on we will simply refer to these PMDC motors as gear heads.

The advantage of using gear head motors is that they are readily available in many sizes, provide a lot of torque for the power consumed, are available with a wide choice of output speeds, come with various voltage ratings, will operate, with reduced speed and torque, over a sizeable fraction of their voltage rating, and are reversible. The main disadvantage is that gear head motors are not precise. That is, two motors of the same model, manufactured on the same day, and operated with identical current and voltages, will NOT turn at exactly the same rate. Thus a robot with two drive motors, the most common configuration, will not move in a straight line without some way of controlling individual motor speeds.

Now let's list some of the more important gear head motor parameters:

- Availability – Gear head motors come in very small to fractional horse power sizes. They are plentiful on the surplus market, which makes them inexpensive.
- Voltage – The typical motor operating voltage for modest sized robots is in the range of six to 24 volts.
- Torque – Typical motor torques vary from 20 oz-in, useable for small platforms, to 80 oz-in, appropriate for eight to ten inch robots, and to several foot pounds, capable of driving robots weighing 50 to 75 pounds.
- Motor Speed (ω) – The shaft RPM combined with the size of the wheels determines the maximum speed of the robot. Typically, wheels for hobbyist and contest robots may vary from two to eight inches in diameter, with the three to five inch sizes predominating. Note we use the Greek letter omega, ω, for motor speed. We will use the letter V for vehicle speed.

Although most gear head motors are reversible, this is not true of all gear head motors and you should check for reversibility in the motor specifications. All motors have a large number of parameters that completely specify their operation. Many of these will not be of high interest to us. For instance, a motor's rotational inertia is rarely of concern for our applications. The most important parameters of interest for us are motor speed, torque and voltage rating. Here's an example of a reseller's ad for a surplus Globe motor (Photo 1) that we will examine in more detail later:

24 Vdc, 85RPM Gearmotor w/Shaft Encoder

Globe Motor #415A374. Powerful little gearhead motor. 85 RPM @ 24 Vdc @ 0.150 Amps (no load). Normal rated load, 80 oz.in. @ 63 RPM @ 0.58 amps. Works fine at lower voltages. 0.25 diameter flatted shaft is 0.75" long. Overall length 3.5" not including shaft. 1.2" diameter *motor. 1.37 diameter gearhead. Three mounting holes (4-40 thread) on motor face, equally spaced on 1.052" diameter bolt circle. 12" wire leads. Encoder information: Red lead – Vcc (24 Vdc Max), Black lead – ground, Blue lead – Channel A. 2 cycles per revolution. Output: Current sinking (no pull up resistor inside encoder.)*

This is the kind of information we like to see. We know the model number, the no load motor speed as well as the motor speed and torque at some point on the operating curve (more about that later), the current consumption, the size, details of the mounting, and the encoder, which can be used to tell us the actual speed of the motor.

Photo 1. *Globe Motor #415A374*

Before we delve further into motors and wheels, we need to quantitatively examine the locomotion requirements. The robot's operating niche informs us of the locomotion requirements, usually speed and torque.

Setting Motor Performance Requirements

A mobile robot goes somewhere, some how. Is it indoors or outdoors, is the terrain level, is high speed desirable, are there obstacles, is precise movement necessary? We must ask ourselves these and other questions in designing the robot locomotion platform. To begin the basic mobility platform we need to decide on the overall size. The motors, wheels and batteries constitute most of the robot bulk and weight. In order to put these units together, we need to scrutinize the contest rules, add in our strategic approach, and derive the platform requirements. The two most basic requirements for robot drive motors are rotation speed (some times called angular velocity) and torque.

We all know what speed is. For motors we measure rotational speed, how fast the shaft rotates in revolutions per minute (RPM), degrees or radians per second being a less common measurement unit. But what is the turning force called torque, how do we measure it, and how do we find out how much our motors need ? There are many things that can create a force. An objects weight is the force of gravity acting on it. Common forces arise from mechanical, electrical and magnetic effects. A stretched or compressed spring exerts an elastic force.

Electrical current moving through a magnetic field generates the force that makes PMDC motors turn. Forces are used to accelerate objects (change their speed or direction). A force acting on a lever generates a torque around the pivot point. More force or a longer lever arm generates more torque. Just as force can be used to change the linear speed of an object, torque can be used to change rotational speed.

Motor torque turns the robot wheels and propels the robot. The robot ground speed will depend on how fast the motor shaft rotates and the diameter of the wheels. If the wheels are not directly mounted on the motor shaft, then there may also be a gear ratio between the motor shaft and wheel axle that needs to be considered. For gear head motors, the motor specs take into account the gearing ratio in the gear box that comes attached to the motor. How fast a motor turns for a given input voltage depends on its load. A free spinning motor, termed a “no load” condition, will rotate faster than a “loaded” motor, one that has to perform work. A heavy robot or one going up an incline imposes more work on the motor. The more work the motor has to do, the slower it turns, and the more electrical current it consumes. As the load is increased, eventually the motor will stop turning or “stall”. A prolonged stall can be very bad for a motor. The motor will be using a maximum amount of current and may over heat, possibly damaging or destroying the motor. The major variables that determine motor speed and torque are the robot weight, the terrain, and the robot speed requirements. In Part III, we will analyze several robot contests and robots that were built for them. For now we want to emphasize, once again, that the motor specs must be derived from the contest environment and the robot task.

Changing Gear Ratios

If the wheel is attached directly to the gear head motor shaft,
then the wheel and motor angular speeds are the same.

If you add additional gearing**,**
then
wheel speed = motor speed / G
wheel torque = G x motor torque
G = gear ratio
= output teeth / input teeth

Larger output gear gives slower more “torquey” robot

There are many different types of contest playing fields, which necessitate different motors. Some playing fields have long stretches, some require a lot of maneuvering, some have lots of stop and go stations, some follow curving paths from point-to-point, etc. For contests that involves a lot of short runs, the following of a curving path, or a constant change in direction, the primary challenge is one of control. As a first estimate of how fast the robot must go, consider the length of the course and the time limit. This gives an average speed just to finish the course. You may want to double this speed as a first goal and pick the motor speed and wheel size accordingly. Since most motors will run at a fraction of their no load speed, you can start slowly and build up.

It is much more common for a contest to have straight runs or open fields for the robot to roam and explore, and we will therefore expend a considerable effort into estimating the motor torque needed to give our robot a commanding presence on the playing field. If the robot is traveling over level terrain, then the torque just needs to accelerate the robot from a dead stop to its “cruising” speed in a short time. Since most contest arenas are of limited extent, we have to make allowances for the robot to speed up, slow down, turn, do something, speed up again, etc. many times during the course of the event. Inclined surfaces, bumps, a small step or playing surface irregularities require extra oomph to overcome.

In the next several pages we will go through the procedure of matching performance requirements and motor specifications in some detail. This material will use some basic physics and algebra. If you are not familiar with these concepts, I suggest you hang in there and get as much as you can from the discussion. The specific examples in Part III will illustrate the motor selection process.

Motor Speed

In this section, we will take a detailed look at the basic relationships between motor speed, wheel size, robot speed and robot performance. This is the first step in giving us the ability to specify and choose motors based on robot performance objectives. Since the speed requirement is easier to estimate, we begin there. Most contests either have a time limit, use speed directly in the scoring or as a tie breaker. The minimum speed requirement can be derived from the contest rules: the size of the playing field, how much of it has to be traversed, the time limit, and the tasks that have to be performed. If the contest has been run in the past, we may be able to ascertain how previous contestants performed and use that information to set a speed objective. As the speed of a robot increases, so does the difficulty in controlling it. Thus, we usually start with the initial goal speed in the testing phase and gradually increase it until the robot performance reaches its limit.

We begin with the relationship between robot speed, motor speed, and wheel size. The basic equation relating robot speed to motor angular speed is:

$$\mathbf{V} = \omega \times \mathbf{R} \qquad \text{equation 1}$$

where

V is the robot speed in inches/sec

ω is the motor angular speed (how fast the shaft turns) in radians/sec

R is the wheel radius in inches

If the wheel is not directly mounted to the motor shaft, then ω is the wheel angular speed, the rotation rate of the motor modified by any gearing interposed between the motor and the wheel (see the inset box on page 6). Choosing practical units, the relationship between wheel size, motor speed, and robot speed is:

$$\mathbf{V} = \omega \times \mathbf{D} / \mathbf{19.1} \qquad \text{equation 2}$$

where:

V = robot speed in inches/sec

ω = motor speed in revolutions/minute (RPM)

D = wheel diameter in inches

19.1 is a conversion factor to make the units consistent

We can turn this equation around to calculate a required motor speed given a desired robot speed and wheel diameter, or we can calculate a wheel diameter to provide a desired robot speed from a given motor speed. These relationships, using the same units of rev/minute and inches, are:

$$\omega = \mathbf{19.1} \times \mathbf{V} / \mathbf{D} \qquad \text{equation 3}$$

$$\mathbf{D} = \mathbf{19.1} \times \mathbf{V} / \omega \qquad \text{equation 4}$$

Speed Requirements

Now we address how to pick a motor speed from the contest description and our performance goal. The contest arena will have certain runs over which the robot is free to move. Let's consider two circumstances to illustrate the process, which can then be adapted to other situations.

First consider a contest that has taken place before. An excellent example is the Trinity College Home Robot Firefighting contest (http://www.trincoll.edu/events/robot/). This contest distributes videos of past events. From these we can time the various robots shown to estimate their speeds. This tells us that the observed speeds are doable and a competitive robot will probably have to be at least as fast. Let's say the average speed of a suitable competitor is V_{old} and we want to go faster by some factor **f**. Then our average speed requirement is simply

$$\mathbf{V_{avg} = f \times V_{old}}$$

And our motor speed requirement is:

$$\boxed{\boldsymbol{\omega} = \mathbf{19.1} \times \mathbf{f} \times \mathbf{V_{old}/D}} \qquad \text{equation 5}$$

Second, for new contests, or ones on which we have no knowledge of prior competitions, we can choose an average speed from a knowledge of the course the robot follows and the contest time limit. This is the minimum speed needed to finish on time, so we will choose a speed that is a factor **f** larger. How much larger depends on how realistically the contest time limit was set and your robot building experience. A factor of two or three is not outrageous. A given motor's speed is adjustable over a large range, and changing wheel diameters can help also, so the initial choice is not crucial.

Let's examine picking an average speed for a contest in more detail. Average speed is just the distance traveled, **X**, divided by the time taken, **T**. Choose a distance appropriate to the contest. Perhaps one of the longer runs, or the distance between objectives, or the whole field if the rules permit. The motor will accelerate over part of the range, **S**, during which the robot goes from zero to some cruising speed, $\mathbf{V_C}$. For loads that are not too great, a motor will achieve a steady speed and torque over a short distance, **S** (we will show how to calculate this in detail later on). The situation is illustrated in Figure 1. The time **T** is divided into two parts, $\mathbf{T_1}$, time of the acceleration over distance **S**, and $\mathbf{T_2}$, the time spend cruising at speed $\mathbf{V_C}$. Then the average speed is:

$$\mathbf{V_{avg} = X / T = X / (T_1 + T_2)} \qquad \text{equation 6}$$

where

X is the total distance traveled

T_1 is the acceleration time

T_2 is the cruising time

The motor speed goes from zero when the motor first starts to its cruising speed, $\mathbf{V_C}$, at time $\mathbf{T_1}$. We will assume that the average speed over **S** is $\mathbf{V_C /2}$, and the average speed over the remainder of the distance, **X – S**, is the constant cruising speed $\mathbf{V_C}$. The corresponding times are the respective distances divided by their speeds, or

Acceleration Distance (S)		**Cruising Distance (X – S)**	
0	$\mathbf{V_C}$	$\mathbf{V_C}$	**Speed**
0	$\mathbf{T_1}$	$\mathbf{(T_1 + T_2)}$	**Time**

Average Speed $\mathbf{= X/ (T_1 + T_2)}$

$\mathbf{V_A = X/[\ V_C/2 + (X – S)/ V_C]}$

$\mathbf{V_C = (1 + S/X) \times V_A}$

Figure 1. Calculation of Average Speed

$$\mathbf{T_1 = S / (V_C /2)} \quad \text{and} \quad \mathbf{T_2 = (X – S) / V_C} \qquad \text{equations 7}$$

Substituting the times from equations 7 into equation 6,

$$\begin{aligned}\mathbf{V_{avg}} &= \mathbf{X / [S / (V_C /2) + (X – S) / V_C]} \\ &= \mathbf{V_C \times X / (X + S) = V_C / (1 + S/X)} \qquad \text{equation 8}\end{aligned}$$

Solving for the cruising speed,

$$V_C = (1 + S/X) \times V_{avg} \qquad \text{equation 9}$$

where

V_C is the robot cruising speed in inches/sec

S is the acceleration distance

X is the cruising distance

V_{avg} is the average desired speed

The motor cruising speed rating appropriate to the task, from equation 3, is

$$\omega_C = 19.1 \times (1 + S/X) \times V_{avg} / D \qquad \text{equation 10}$$

where

ω is the cruising motor speed in revolutions/minute (RPM)

V_{avg} is the average desired speed in inches/sec

D is the wheel diameter in inches

S is the acceleration distance

X is the cruising distance

Motor Torque

To pick an appropriate motor, we need to know how strong it is in addition to how fast it turns. The measure of "strength" we want is the motor torque, the motor's ability to push the robot along. Estimating the required torque is more difficult than estimating the necessary motor speed. Before we get into calculating motor torque from performance requirements, let's make some estimates of the minimum and maximum useable torque.

Frictional Forces

Friction determines the minimum force required to move a robot from a dead stop and this determines the minimum motor torque required to move a robot at all. Friction is a force than opposes the motion between two surfaces in contact with one another. It always acts in a direction opposite to the motion. The amount of force that it takes to begin sliding depends on static friction. Once the sliding begins, the frictional force decreases slightly and is called dynamic friction. For the most part, our robots experience a third type of friction called rolling friction or more commonly rolling resistance, which is considerably smaller than sliding friction. Rolling resistance is caused by the deformation of the tire and surface, and depends on the tire and surface materials. For any type of friction, the coefficient of friction is the ratio of the frictional force to the weight pressing on the surface, called the normal (or perpendicular) force, see Figure 2.

$$\mathbf{C = F_f / F_N} \qquad \text{equation 11}$$

where

C is the coefficient of friction

F_f is the frictional force to begin motion

F_N is the normal force

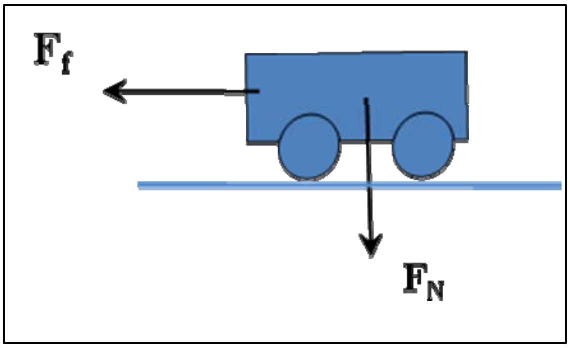

Figure 2. Frictional Forces Diagram

On a level playing surface, the normal force is just the robot weight, W. In order to propel the robot, the motor torque must at a minimum overcome the external torque of the friction force acting on the radius of the wheel. Thus the minimum required motor torque is:

$$\mathbf{T = F_f \times R = C \times F_N \times R = C \times W \times R}$$

Converting units,

$$\mathbf{T = 8 \times C \times W \times D} \qquad \text{equation 12}$$

where:

T is the torque in oz-in

C is the coefficient of friction

W is the weight in lbs

D is the wheel diameter in inches

We must be careful to distinguish rolling friction, C_R, from static friction, C_S. Typically, rolling resistance varies from 0.001 for steel on steel to 0.030 for a bus on asphalt. While the coefficient of rolling friction is usually very low, the coefficient for static friction can be quite large, even greater than 1. As an example, for a $C_R = 0.03$, the minimum torque to move a 10 lb robot with 3 inch diameter wheels would be:

$$\mathbf{T = 8 \times 0.03 \times 10\ lbf \times 3\ in = 7.2\ oz\text{-}in}$$

This is a pretty puny motor but should just keep a robot rolling. Because of various inefficiencies in all the components, we'd use two of these small motors to propel our robot, but this would still be the bare minimum. The robot should cruise fine but will take a long time to come up to speed and any surface irregularity could cause it to stall.

Another illustrative calculation is the maximum useable torque for acceleration. This would be the torque that causes the wheels to exert a force in excess of that supported by static friction, resulting in wheel slippage. Making a similar calculation as above but using a coefficient of static friction equal to 0.7, the maximum torque before slipping is

$$\mathbf{T = 8 \times 0.7 \times 10\ lbf \times 3\ in = 168\ oz\text{-}in}$$

Even a pair of hefty 80 oz-in motors used at full capacity would still be marginally safe. Between these two extremes, 7.2 oz-in and 168 oz-in, lies a wide choice in motor torque values.

One method for measuring the coefficient of static friction is to use a small weighing scale to find the force necessary to just drag the robot along a flat surface. The robot wheels must be locked in place, tapping them together or to the chassis works well. Then dividing the dragging force by the robot weight gives an approximate value of the coefficient of static friction. In the section below on playing fields with inclined planes, we'll give another experimental method for finding the coefficient of static friction.

Although we want to know how to estimate the motor torque necessary to achieve our cruising speed, these friction limits give us useful boundaries. Now it's time to turn to the most common type of playing field.

Level Playing Fields

On a level playing field we are primarily concerned with the torque needed to overcome the robot's inertia (i.e. mass) in order to accelerate it to a desired speed. This is a little messy to calculate. We will go over the basics for those who are interested and conclude with some practical guidelines. How fast a robot changes its speed is called acceleration and it depends on the net driving force and the weight of the robot, given that the acceleration is not so great that the wheels lose traction and slip. On smooth or slick surfaces especially, the acceleration may be limited by wheel slippage but this is not usually a problem. If it is, the situation can be remedied either with higher traction wheels or by ramping up the robot speed gradually by increasing the applied motor voltage in steps.

The robot acceleration is given by Newton's second law of motion:

$$\mathbf{F} = \mathbf{m} \times \mathbf{a} \qquad \text{equation 13}$$

where

F is the net accelerating force.

m is the mass of the object that the force acts on

a is the resulting acceleration

The net force is a combination of the force supplied by the motor minus any other forces acting on the robot. For level surfaces, the other force is mostly friction, of one kind or another. Since rolling frictional forces are small, we will neglect them, especially since we will be generous in deriving the motor torque requirements. Playing fields with inclines have significant downhill gravity forces and are treated in the next section.

Applying equation 13 to the force of gravity:

$$\mathbf{W} = \mathbf{m} \times \mathbf{g} \qquad \text{equation 14}$$

where

W is an objects weight

m is the object mass

g is the acceleration caused by gravity

Equation 14 gives us a way to find an object's mass by weighing it. Substituting the mass from equation 14 into equation 13 gives:

$$\mathbf{F} = \mathbf{1.33} \times \mathbf{W} \times \mathbf{a / g} \qquad \text{equation 15}$$

where
F is the force in ounces needed to accelerate the robot
W is the robot weight in pounds
a is the acceleration in inches/sec-sec
g is the acceleration of gravity = 32.2 feet/sec-sec

We use this peculiar set of mixed units for the convenience of weighting the robot in pounds, measuring force in ounces, and measuring modest accelerations in inches/sec-sec.

The force on the playing surface that accelerates the robot is generated by the motor torque turning the wheels. Since torque is equal to force times the lever arm it acts through, the required motor torque is simply:

$$\mathbf{T} = \mathbf{F} \times \mathbf{R} = \mathbf{F} \times \mathbf{D / 2} \qquad \text{equation 16}$$

where
T is the torque in oz-in
F the force exerted by the motor in ounces
R the wheel radius in inches
D the wheel diameter in inches

Finally, substituting the force required to produce an acceleration from equation 15 into torque equation 16 and using the value for the acceleration of gravity, we have,

$$\mathbf{T} = \mathbf{D} \times \mathbf{W} \times \mathbf{a / 48.4} \qquad \text{equation 17}$$

where
T is the motor torque in oz-in
D is the wheel diameter in inches
W is the robot weight in pounds

a is the robot acceleration in inches/sec-sec

Equation 17 confirms what we expect from common sense. It takes more torque to move a robot that is heavier and/ or has bigger wheels. Now we can estimate how much motor torque is needed once we know the robot's weight, wheel diameter and the desired acceleration.

<u>Acceleration</u>

Acceleration changes the robot speed, so finding the needed acceleration amounts to deciding how much time it takes the robot to go from a dead stop to some cruising speed. Decelerating the robot is usually not an issue. There are several effective decelerating options available. Just cutting the power makes the robot work against the motor gear ratio, which is a pretty good brake. For emergencies, one can throw the robot into reverse for a very brief time. While this is effective, it's not recommended, since, for one thing, it's very hard on the gears. A third option, dynamic braking, is possible with some electronic speed controls.

Now onto estimating the required motor torque. Knowing a speed either from the contest arena set-up, or from competitors we wish to better, we can make a logical determination of the motor torque needed to accomplish the task.

There is a standard kinematic equation that relates the cruising speed, V_C, attained by a constant acceleration from a stop, **a**, over an acceleration distance, **S**. Namely

$$\mathbf{V^2_C = 2 \times a \times S \quad or \quad a = V^2_C / 2 \times S} \qquad \text{equation 18}$$

At last, we can express the acceleration in terms that we can measure and experiment with. Substituting V_C from equation 16,

$$\mathbf{a = [(1 + S/X) \times V_{avg}]^2 / 2 \times S} \qquad \text{equation 19}$$

We can simplify equation 19 by substituting $[(1 + S/X) \times V_{avg}]$ from equation 10 into equation 19,

$$\mathbf{a = [\omega_C \times D / 19.1\,]^2 / 2 \times S} \qquad \text{equation 20}$$

With the acceleration finally in hand, we return to equation 17

$$\mathbf{T} = \mathbf{W} \times \mathbf{D} \times \mathbf{a} / \mathbf{48.4}$$

to find the motor torque required to produce this acceleration. Substituting the acceleration from equation 20 into equation 17,

$$\mathbf{T} = [\omega_C \times \mathbf{D} / \mathbf{19.1}]^2 \times \mathbf{W} \times \mathbf{D} / \mathbf{96.8} \times \mathbf{S}$$

$$\boxed{\mathbf{T} = \omega_C^2 \times \mathbf{D}^3 \times \mathbf{W} / \mathbf{35{,}314} \times \mathbf{S}} \qquad \text{equation 21}$$

where

T is the torque is oz-in

ω_C is the motor speed in rev/min at cruising speed V_C

D is the wheel diameter in inches

W is the robot weight in pounds

S is the acceleration distance in inches

Equation 21 is very instructive. Here we see how the torque is related to the fundamental parameters of robot weight, wheel size, motor speed, and also to acceleration distance. Equation 21 gives the constant torque needed to accelerate to cruising speed in a distance S. The torque in equation 21 is an average torque over the acceleration distance, which is usually short. Looking at equation 21 you may be concerned about how to choose S. For now we are using the playing field description and a robot navigation strategy to develop motor requirements. In Part III we will use the behavior of PMDC motors to find out how to calculate S and how to determine what a motor can deliver. We will also illustrate these techniques with several worked examples. For now, we turn to a different type of playing field.

Playing Fields with Inclines

Figure 3 illustrates the situation for a robot on an inclined plane. Part of the weight of the robot presses it against the surface, called the normal force $\mathbf{F_N}$, and part is directed down hill, called the tangential force $\mathbf{F_T}$. As common experience tells us, the steeper the incline, the harder it is to drive the robot up hill. For those not familiar with vectors and trigonometry we give the results here and explain how to use the results later.

The force to maintain a robot's position on an incline or to move the robot uphill at constant speed, is simply equal to the portion of its weight directed downhill, that is the tangential force,

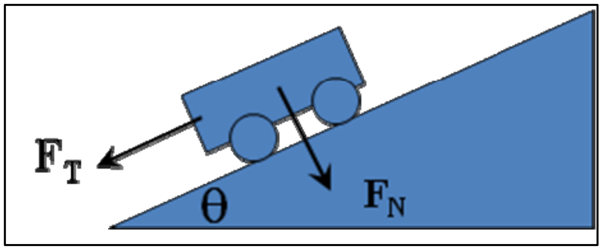

Figure 3. Gravitational Forces For A Robot On An Incline

$$\mathbf{F_T = 16} \times \mathbf{W} \times \mathbf{\sin(\theta)} \qquad \text{equation 22}$$

where

F_T is the force in oz

W is the robot weight in pounds

θ (theta) is the inclination angle (tilt) of the surface

Sin () is a trigonometric function (pronounced "sign")

The force pressing on the incline is equal to the portion of the robot's weight against the plane, the normal force,

$$\mathbf{F_N = 16} \times \mathbf{W} \times \mathbf{\cos(\theta)}$$

where

F_N is the force in oz

W is the robot weight in pounds

θ (theta) is the inclination angle

Cos () is a trigonometric function (pronounced "co-sign")

When the gravitational tangential force pulling the robot down the incline exceeds the static friction force, $\mathbf{f_S}$, sliding will occur. As the steepness of the incline increases from zero, a point will be reached when the forces are in balance. That is when,

$$\mathbf{f_S = F_T}$$

As the slope increases further, the robot will begin to slide. Since the coefficient of sliding friction is less than the coefficient of static friction, the robot continues to slide. Using equation 11, we can calculate the coefficient of static friction, $\mathbf{C_S}$, from the slope when the robot begins to slide.

$$\mathbf{C_S = 16} \times \mathbf{W} \times \mathbf{\sin(\theta) / 16} \times \mathbf{W} \times \mathbf{\cos(\theta)}$$

$$\boxed{\mathbf{C_S = \tan(\theta)}} \qquad \text{equation 23}$$

where

C_S is the coefficient of static friction

θ is the inclination angle

Tan () is a trigonometric function (pronounced “tangent”)

The procedure for measuring $\mathbf{C_S}$ is fairly simple. You lock the wheels of the robot (taping them together works) so they cannot turn, place it on an inclined surface (of the same material as the contest playing field) and gradually raise one end, increasing the inclination angle until the robot begins to slide down hill. Then the downward acting weight of the robot is just enough to overcome the restraining frictional force (this method doesn’t work well for rolling friction since the wheels are restrained from moving freely by motor and gears). The various trigonometric functions such as sin, cos and tan can be found in mathematical tables or is easily obtained from a scientific pocket calculator. The Windows operating system for PC’s also has a calculator under the Accessories programs accessible form the Start menu – Start/Programs/Accessories/Calculator. Choose the Scientific mode.

To find the torque needed to overcome the pull of gravity down the incline, we simply multiply the tangential force, equation 22, by the wheel radius.

$$\boxed{\mathbf{T = 8} \times \mathbf{W} \times \mathbf{D} \times \mathbf{\sin(\theta)}} \qquad \text{equation 24}$$

where

T is the motor torque in oz-in

W is the robot weight in pounds

D is the wheel diameter in inches

θ is the inclination angle

To propel the robot uphill, you will need to provide enough additional torque to overcome forces and inefficiencies not included in this simple treatment. Starting on an incline with too much torque may result in wheel slippage since the normal force, $\mathbf{F_N = 16}$ x $\mathbf{W}$ x $\mathbf{cos(\theta)}$, decreases as the slope increases, thereby lessening the static frictional force that keeps the wheels from skidding.

Now we have all the information and relationships we need to set the motor requirements from a description of the robot's niche – environment and task. Next we need to know more about PMDC motors. Specifically, we need quantitative relationships between the motor speed, torque, and current.

PART II. PMDC Motor Operation and Specification

Operation

The basic physics that governs the operation of an electric motor was discovered by Hans Christian Oersted in 1820 when he noticed that a current in a wire deflects a magnetic compass needle. The current carrying wire produced a mechanical force on the compass needle. In a PMDC motor, a coil of wire is wrapped around a rotating spindle called the armature, which is surrounded by a permanent magnet split into north and south poles on either side, Photo 2. When current is passed through the armature windings, a force is produced in the wire and thence to the armature, which causes it to turn in the stationary magnetic field. In order to keep the armature turning, the current must change directions as the one side of the windings pass from the north to south magnetic poles (or vice versa). This is done by splitting the winding into many parts, each of which is connected to an electrically isolated metal bands at one end of the armature. These bands, together with stationary "brushes" that are spring loaded to ride on the bands, form a rotating switch called a commutator. As the armature and commutator turn, the current in the individual wire coils changes direction as each coil changes from one magnetic pole to the other. Thus a continuous force keeps the motor turning in the same direction.

When a motor is first turned on, the shaft is not rotating and the motor is at its stall point. It momentarily draws stall current, $\mathbf{i_s}$, with stall torque, $\mathbf{T_s}$. As the motor begins to turn, the motor speed increases, the current decreases, and the torque decreases as the motor approaches its equilibrium operating point, ($\mathbf{i_p}$, $\mathbf{T_p}$, $\boldsymbol{\omega_p}$). The equilibrium point is that at which the motor output torque is equal to the load on the motor shaft. For our robots this load is created by rolling friction, uphill travel, or other forces, e.g. pushing something

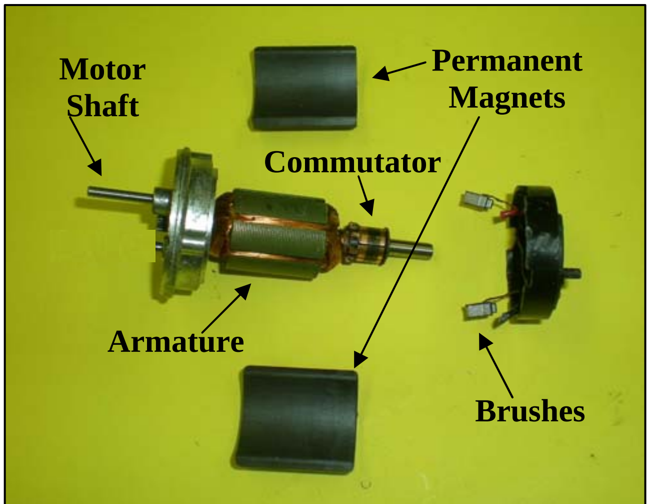

Photo 2. Globe Motor Disassembled

It is useful to consider some relationships between motor torque, speed, and current consumption. As a rule, we will not be concerned with power consumption or efficiency since contest time limits are usually too short for this to be a concern.

A motor with no external load (zero torque), operating at its nominal rated voltage, will spin at its maximum rate, the no load speed $\boldsymbol{\omega_o}$. At the other extreme, there is some external load that will exceed the maximum torque the motor is capable of and the motor will stall. In between these extremes the motor speed is a linear function of its torque, that is, as the load torque increases, the angular speed decreases. The relationship, illustrated in Figure 4, is:

$$\omega = \omega_o \times (1 - T/T_s) \qquad \text{equation 25}$$

or equivalently,

$$T = T_s \times (1 - \omega/\omega_o) \qquad \text{equation 26}$$

where
ω is the angular speed
ω_o is the no load speed
T_s is the stall torque
T is the torque at ω

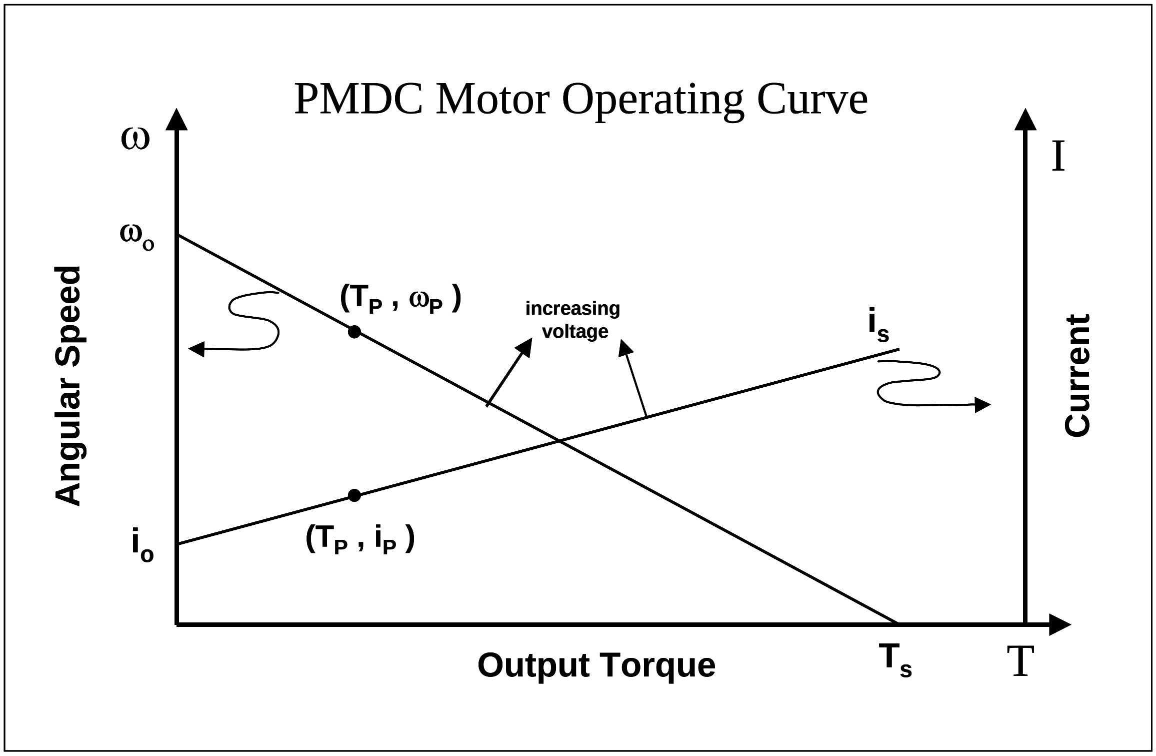

Figure 4. There is a linear relationship between a motor's torque and speed, and between its torque and current. A free running motor, no external load, has a no load speed ω_o, its maximum turning rate, and draws a no load current of i_o. As an external load, T_p, is applied, the motor slows down and draws more current, i_p, as it adjusts its output torque, T_p, to meet the load. If the load is not too great, the motor will continue to run at a new, lower speed, ω_p. If the motor cannot overcome the load, it will stall, that is cease to turn, and draw a stall current, i_s, determined by the resistance of its windings. A stall occurs when the imposed, external torque is equal to or greater than the motor's stall torque, T_s. As the voltage applied to a motor increases, the operating lines shift upward, increasing the no load speed and the stall torque

As the motor operates with increasing load, the current consumption also goes up. The current draw is also a linear function of torque. It starts with a no load value $\mathbf{i_o}$ and increases to some maximum at motor stall. The appropriate relation, also illustrated in Figure 4, is:

$$\mathbf{i = i_o + (i_s - i_o)} \times \mathbf{T / T_s} \qquad \text{equation 27}$$

where

i is the motor current

i_o is the no load current

i_s and T_s are the stall current and torque

T is the torque at current i

Suppliers may say that a motor is “hefty”, “good for robotics”, or some other qualitative statement. This is insufficient information and should be viewed with considerable skepticism. The minimum information we need to intelligently choose a motor, is the operating voltage and no load speed together with a stall torque or a speed at some specified torque. Information about the current consumption is also handy. Let’s work a specific example.

For the Globe motor described in Part I, we can use equation 26 to find the stall torque, T_s. Rearranging terms and substituting the known speed and torque values,

$$\mathbf{T_s = \omega_o \times T / (\omega_o - \omega) = 85 \times 80 / (85 - 63) = 309\ oz\text{-}in}$$

That’s a lot of torque, however, the motor is not spinning! Now that we know T_{stall}, we can calculate any motor speed for a specified torque, or a torque for some desired motor speed. Substituting the stall torque and operating current values into equation 27, we can find the stall current.

$$\mathbf{i_{stall} = (i_s - i_o) \times T_{stall} / T_s + i_o = (0.58 - 0.15) \times 309 / 80 + 0.15 = 1.81\ amp}$$

How does this compare with reality? With the actual motor in hand, I ran two checks, one by using Ohm’s law, and another by measuring the stall current directly. Using a multimeter we can measure the motor coil resistance. For the Globe motor it’s 13.7 ohms. Ohm’s law gives:

$$\mathbf{I = V / R = 24 / 13.7 = 1.75\ amp}$$

To measure the stall current directly, I put the motor in a vise and held the shaft with a pair of locking pliers to prevent it from turning. Applying 24 volts from a battery supply and measuring the current with a multimeter gave a stall current of 1.75 amps. This measurement is consistent with the Ohm’s law calculation. The difference between the two measurements and the calculation based on stall torque, is minor, about 3%, and is due to the expected variation in motor-to-motor specs.

One further note, the operating point given in summary motor specifications is usually around the maximum efficiency point. This is very roughly at from 70 to 80 % of the no load speed and from 20 to 30% of the stall torque. From the above numbers, the 63 RPM, 80 oz-in torque point lies at 63/85 = 74 % of the no load speed and at 80/309 = 26% of the stall torque. The maximum power delivery of a motor is at one half the no load speed, which is also at on half of the stall torque and one half the stall current.

Gear Head Motor Specifications

Generally I look for certain minimum specs before purchasing a motor, allowing plenty of margin to accommodate a developing platform. The minimum specs I like to see are voltage rating, no load motor speed, motor speed at some specified torque value, and current draw. If you know the motor manufacturer and motor model number, you may be able to find the manufacturer's original specs on-line through one of the commonly available Internet search engines.

1. Voltage Rating. Although motors come with a large variety of operating voltages, we need to choose one that corresponds to the batteries we will be using. There is more variety and availability in 12 and 24 volt motors, and these are also very convenient for use with standard battery packs and are the right physical size for robots measuring from eight to 16 inches in overall size. For the larger robots, say 16 inches or so in diameter, you will probably use sealed lead acid batteries, since these have a lot of capacity for driving heavy robots. For smaller robots, from 8 to 12 inches in diameter, NiMH batteries are often used. In addition to the standard AA, C and D sizes, these come in convenient, pre-made battery packs of from 3.6 to 48 volts. Battery packs made for remote controlled cars come in 7.2, 8.4, and 9.6 volt packages that are widely available and convenient to use. For 12 v. motors I use either a couple of 7.2 or 8.4 volt, or a single 9.6 volt pack, depending on whether or not the motor will be used up to, below, or above its rated voltage. For 24 v. motors, I use three of the 9.6 volt packs in series. However, you should feel free to use any combination of battery types or custom packs available and convenient.

2. Motor Speed. As mentioned before, for a given applied voltage, ω varies with the motor load. As the load on the motor increases, the current draw increases, and ω decreases.

Changing the voltage shifts the motor operating curve as shown in Figure 4. This is the primary method used to control the speed of a robot. As the applied voltage is lowered, the speed vs. torque curve is lowered proportionally. The motor speed rating we are interested in is the value at the torque we need. We can calculate a (motor speed, torque) point from the motor specs, but where do we begin, with the RPM or the torque? In Part III we give a procedure that starts with some assumptions and then iterates based on the available surplus motors. The speed we are really interested in is that of the robot and, as have seen, the robot speed depends on the wheel diameter. As a rough estimate, wheel diameters will be in the range of two to eight inches, depending in part on the overall size of the robot, and the no load motor speed will be in the range of 40 to a few hundred RPM.

3. Torque. The motor torque, acting through the wheels on the playing field, determines how fast a robot can accelerate, how steep an incline it can climb, how much load it can carry, or with what force it can push or pull. For good measure we generally oversize the motor torque to allow for unknowns that arise as the robot platform develops, especially for an inevitable increase in the robot weight, and for an increase in performance as we push the design. After making a best estimate torque estimate, I always double it, although a 50% increase is probably sufficient. How much torque an *individual* motor needs depends in part on how many drive motors the robot uses. Typically there are two independent motors in most robot drive configurations. If a dual drive motor platform needs, for instance, 60 oz-in of torque, then two such motors gives the needed reserve torque with gusto. Other drive configurations may use one or four motors and we can scale the total platform torque accordingly

4. Current Draw

The motor voltage times the current it uses equals the power the motor consumes. The torque times the rotational velocity the motor produces is the power the motor supplies. As the load on the motor increases at a given operating voltage, the motor slows down. This allows it to draw more current and thereby increase the output torque to meet the challenge of the increased load. Looking at motor specs, we can usually judge if a motor is too big or small for our application by looking at the current draw. If the motor only demands 0.01 amps at no load, it's too small for most robot applications. On the other hand, if the idling current draw is 1 amp, it's probably too big. The amount of current a motor uses will determine how long it can operate before draining the batteries.

5. Physical Measurements

Most contests have a specific rule on maximum allowable robot size. Even if there were no rule limitation, there are always practical limitations. This size limitation imposes a limit on the size motor we can use. It has to physically fit on the platform. Many robots use two motors that are placed opposite each other on the platform. If the platform is eight inches across, the max size of the motor is 4 inches long, including the motor shaft. In practice, there is also the wheel thickness to consider and some space between the back of the motors may be needed to connect the power. Motors with encoders are longer than those without. All this needs to be taken into account. In addition, the shaft may come in a size that makes it difficult to mount a wheel. Take note that some motors have English and some metric sized shafts and mounting screw sizes.

6. Special Features

Several times we have mentioned motor encoders without describing them. Motor encoders indicate how many times a motor shaft turns. The encoding unit is usually attached to the back of the motor where an extension of the motor shaft turns inside the encoder generating a signal that can be used to indicate the motor speed. The most common types of encoders use an optical or magnetic sensor to measure the shaft rotation. The encoder has a separate, usually lower, voltage connection and generates a series of pulses that can be counted with a microcontroller or special circuit. An important encoder spec is its resolution or how many pulses it generates for each turn of the motor shaft. Motors may come with other goodies that may or may not be useful. Some of these are brakes, clutches, right angle gear heads, special mounting brackets, or output shafts on both sides of the motor.

PART III. Motor Selection Procedure and Examples

Now we are ready to develop a procedure for selecting motors for particular applications. We will use the contest description and our performance goal to estimate the motor characteristics needed and then choose a motor and use its actual specifications to check whether it is adequate. If not, we will adjust our estimate and look for a different motor or use a different strategy. The general procedure is given below.

Procedure

STEP 1. Motor Requirements. From the playing field layout and the contest rules, we find the motor speed and torque to meet our speed goal. To do this we need to pick an initial wheel size and guess at an acceleration distance. We can refine these picks later if need be. The motor rotation speed to achieve the cruising speed we desire is given by equation 10:

$$\omega = 19.1 \times (1 + S/R) \times V_{avg} / D$$

The torque required to accelerate to the cruising speed is given by equation 21,

$$T_{acceleration} = \omega^2 \times D^3 \times W / 35{,}314 \times S$$

In the case of inclined playing fields, this acceleration torque is added to the station holding torque is given by equation 24:

$$T_{incline} = 8 \times W \times D \times \sin(\theta)$$

STEP 2. Motor Specifications. Now we search for a motor with the speed and torque requirements determined by the equations in Step 1 and note its published specifications. From equations 25, 26, and 27, which we will refer to as the motor equations, we can rearrange terms as necessary to convert the specifications for available motors to their actual values of motor speed and torque.

MOTOR EQUATONS

$$\omega = \omega_o \times (1 - T/T_s)$$

$$T = T_s \times (1 - \omega/\omega_o)$$

$$i = i_o + (i_s - i_o) \times T / T_s$$

STEP 3. Motor Operation Point. The cruising operating point for level playing fields is, from equation 12:

$$\mathbf{T_p = 8} \times \mathbf{C} \times \mathbf{D} \times \mathbf{W}$$

For rolling friction, the value of **Cr** is difficult to determine, however, from published tables for many materials on many surfaces, we will choose a value of **Cr** = 0.03. For static or dynamic (sliding) friction, we will use values measured for a particular robot on the surface of interest.

For inclined playing fields, we add to the above, from equation 24, the torque to maintain a position on an incline,

$$\mathbf{T_p = 8} \times \mathbf{W} \times \mathbf{D} \times \mathbf{\sin(\theta)}$$

The rotational speed that corresponds to the cruising equilibrium torque is, from equation 25:

$$\omega_p = \omega_o \times (1 - T_p / T_{stall}) \qquad \text{equation 28}$$

Since the torque for cruising, T_p, is much less than that needed for acceleration, the motor will have no difficulty achieving it.

STEP 4. Cruising Speed. The robot cruising speed at the equilibrium point may be found from equation 2:

$$\mathbf{V} = \omega_p \times \mathbf{D / 19.1} \qquad \text{equation 2}$$

Check that this speed corresponds, at a minimum, with the average speed, $\mathbf{V_{avg}}$, in Step 1.

STEP 5. Acceleration Distance. In Part I we put off a discussion of estimating the acceleration distance, S. It's now time to address this. The distance the robot travels from a dead stop to its cruising speed is a complicated function of several variables. The derivation involves some calculus and is given in Appendix I. Here we state the results:

$$\mathbf{S} = (\mathbf{W} \times \mathbf{D}^3 \times \omega_o^2) / (\mathbf{17{,}600{*}T_s}) \times \{\mathbf{Ln}[1/(1 - \omega_p/\omega_o) - \omega_p/\omega_o]\} \qquad \text{equation 29}$$

Equation 29 takes into account the way in which speed and torque change as a motor goes from zero to its cruising speed. Although it appears daunting, we will show its use and utility in the following examples.

If the value of S computed in equation 29 is close to or less than the value used in Step 1, we are finished with the motor selection procedure. If not, we can refine our calculations beginning over with Step 1 and the new value of S. The same motor may still be adequate, or we may need to choose a different motor.

Examples

Finally it's time to work a few motor selection examples for gear head motors. To make the exercise interesting and robust, we'll look at three very different robots that were designed, built, tested, and entered into contests. The first robot travels on a level playing field, the second on a playing field with steep ramps, and the third robot is confined to travel along a rail.

Example 1 - Phoenix: A High Torque, High-Speed Robot

In the Spring of 2000, the southeast division of the Institute of Electrical and Electronic Engineers (IEEE) held its annual conference in Nashville, TN. IEEE is an international association of professional engineers with six divisions in the US. The southeast division, Region 3, holds a robot hardware competition each year at its annual spring convention, SoutheastCon (see http://www.southeastcon.com/ for information on past and present competitions). Electrical engineering students at the University of Alabama in Huntsville built a robot named Phoenix for this competition. The name Phoenix was chosen the week before the contest, after a late night testing session in which a short circuit burned out the motor control electronics. Photo 3 shows Phoenix on a test playing field. Each contest round was a dual between two robots on the field at the same time. Each was given twelve, 7/16 inch diameter steel balls to drop into the nine cylinders in three minutes. The scoring value of each cylinder changed dynamically during the contest so that the robot did not know in advance which cylinders where worth 200, 100 or just 10 points. This was determined by a combination of flashing LEDs and varying magnetic fields under the cylinders that changed during the course of the contest. Because of this randomness and the symmetry of the playing field, the Phoenix robot used a fast sampling strategy that attempted to visit and test as many cylinders as possible in three minutes. As the robot visited and docked with each cylinder in turn, it made measurements to determine the scoring points for depositing a steel ball. If the score was high, a ball was put into the cylinder; if not, the robot visited the next cylinder. At any one time, three of the nine cylinders had high point values and whenever a ball was dropped into one of those cylinders, it assumed a low value and another, randomly chosen, cylinder assumed the previous cylinder's point value. Phoenix circled the playing field, visiting all nine cylinders in each pass around the table.

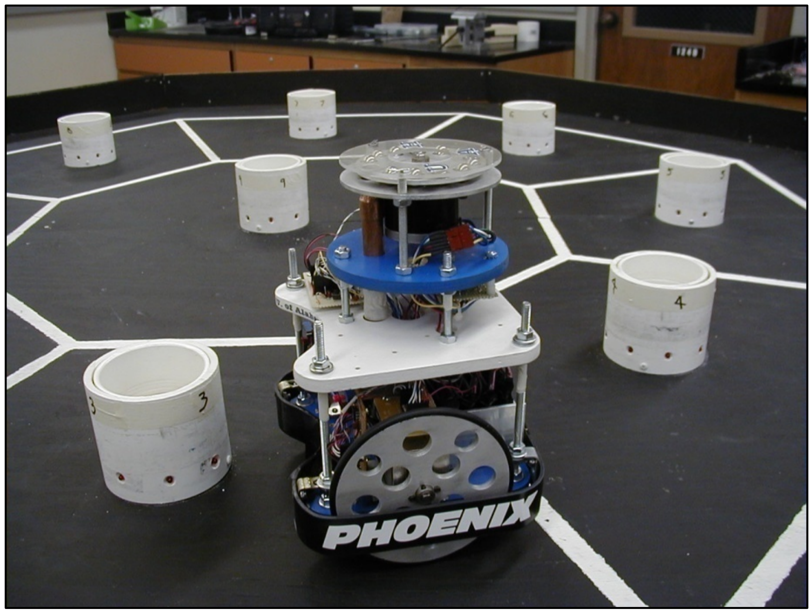

Photo 3. Phoenix on the SoutheastCon 2000 Hardware Competition Playing Field

The strategy for a fast robot requires motors with high motor speed and enough torque to accelerate quickly. One of the consequences of high speed is that the robot is more difficult to control. Phoenix made up for this by having a docking bumper. If the approach to the cylinder was not well aligned, the force of the impact assured that the robot bumper would square itself with the cylinder. In practice this worked quite well.

The contest rules specified a maximum robot size of 20 x 25 x 30 cm high (just under 8 by 10 inches and 12 inches high). This is about what would be chosen anyway given the systems the robot needed for the task – an LED detector, a magnetic field detector, a method to find the cylinders, and a ball dispenser, in addition to the drive and power systems. For maneuverability the robot would use two drive motors in the common differential drive configuration. Given the playing field layout and the strategy, the question is, how fast does the robot need to be? The distance around the playing field, from cylinder-to-cylinder, including visiting the central cylinder, is about 17 feet. In one circuit of the field a robot may expect to deposit three to six balls, depending on the random placement of the high value cylinders and the operation of the opponent. To conservatively deposit twelve balls would then take maybe four circuits. We want to win so we'll design for six circuits in three minutes. Six circuits in three minutes is an average speed of 6.8 inches/sec. However, the robot has to stop to check the cylinder and maybe deposit a ball, so that half of the time it won't be moving at all. To make up for this, the robot speed goal was doubled to 13.6 in/sec.

Step 1. Motor Requirements. First we calculate the motor requirements. Phoenix had a wheel diameter of 5 inches. Picking V_{avg} = 13.6 in/sec as a goal from the discussion above, we have, ignoring the S/R term:

$$\textbf{Motor Speed} = 19.1 \times V_{avg} / D = 19.1 \times 13.6 / 5 = 52 \text{ RPM}$$

Phoenix weighed about ten pounds with batteries. To evaluate the torque needed to accelerate to $\mathbf{V_{avg}}$ we are faced with the difficult decision of choosing S. The distance between cylinders was about 20 inches so we picked a very aggressive S = 1 inch. Whether the S value we pick is realistic or not is difficult to determine at this point. We will return to this choice in Step 5. Putting in all the values, we have:

$$\textbf{Motor Torque} = \omega^2 \times D^3 \times W / 35{,}314 \times S = 95.7 \text{ oz-in}$$

Motor Requirement: Torque = 96 oz-in, Speed = 52 rpm

Step 2. Motor Specs. Looking through a lot of surplus motor specs, the motor chosen for Phoenix was the Globe motor described in Part I with a no load speed of 85 RPM and a speed of 63 RPM at 80 oz-in of torque. From these values we calculate a stall torque of 309 oz-in. In summary, for the Globe motors:

$$\omega_o = 85 \text{ rpm}, \quad T_s = 309 \text{ oz-in}, \quad Cr = 0.03, \quad W = 10 \text{ lbs}, \quad D = 5 \text{ inches}$$

Using the motor equations and the 96 oz-in torque requirement, the Globe motor provides a speed of 58 RPM, a very good match to the motor requirement!

Step 3. Motor Operation Point. The torque needed to maintain Phoenix's speed against the opposing force of friction is only 12 oz-in. Note that a much higher torque, 96 oz-in, is required to achieve that speed. The corresponding cruising speed motor speed is 82 RPM. This is higher than the previous estimate of 52 RPM based on the average speed because the low rolling friction lets the motor run faster.

Operating Point: Torque = 12 oz-in, Speed = 82 rpm

Step 4. Cruising Speed. The higher motor speed gives a new cruising speed of 21 inches/sec. So, if desired, the Globe motors can exceed the initial speed requirement. This indicates that these motors are sufficient to the task and have plenty of reserve for increased performance.

Step 5. Acceleration Distance. Using the torque available for both motors, a stall torque of 309 oz-in each, equation 29 gives an acceleration distance S = 2.8 inches. Previously we assumed S = 1 inch to get a guesstimate at the torque needed. This calculation shows that the motor chosen is capable of meeting and exceeding the speed requirement with an acceleration distance of 2.8 inches. In a distance of 2.8 inches Phoenix achieves a speed of 21 inches/sec, for an average speed of 18.4 in/sec, which is better than our goal of 13.6 in/sec. Therefore, we can run it at a lower voltage and achieve our original desired average speed. We can also run it faster, however, control will at some point become an issue. Phoenix didn't worry about decelerating when it reached a cylinder, it just crashed into it. It's bumper performed the deceleration. Keeping in mind that our assumed coefficient of rolling resistance is a rough estimate, the present analysis tells us that our motor choice can do better than we require.

Being fanatical, Phoenix used two Globe motors, which doubles the available torque. Furthermore, a full charge on the batteries gave about 30 volts. This increases the motor torque by about 25%. Therefore the total torque available with both motors was about 200 oz-in. Since there is plenty of reserve, the final speed and acceleration can be adjusted by varying the voltage applied to the motors.

When the motors were run at full voltage, the wheels slipped a little. As a check on the maximum useful torque, the coefficient of friction for Phoenix was measured at $\mathbf{C_S} = 0.5$. The torque at which we expect wheel slipping is then, from equation 12:

$$\mathbf{T = 8 \times 0.5 \times 10\ lb \times 5\ in = 200\ oz\text{-}in}$$

The torque at which slipping occurs agrees with observation. This is an amazing correspondence, especially considering that the measurement of the friction coefficient is only approximate. Since slipping makes control more difficult, the acceleration was ramped up under software control from zero to about 80% of the max available. Even though Phoenix didn't need all the available torque to accelerate, it was useful in pushing opponents out of the way. Phoenix worked so well and consistently that it won first place among the 34 competing robot entries.

Example 2 – WHIZard: A Fast, Hill Climbing Robot

WHIZard was another design class robot (the WHIZard name was supposed to convey the idea of a robot that was fast and smart). It was built for the 2001 SoutheastCon IEEE hardware competition, which was held at Clemson University in Clemson, SC. The four by ten foot playing field and the WHIZard robot is shown in Photo 4. The ramp angle is approximately 19 degrees. The goal was to pick up ½ inch diameter steel balls placed in ½ inch deep holes in known positions along the playing surface and deposit them in a scoring bin. There were 15 balls in all: six on the home field side, worth 10 points each, three on the flat surface between the ramps, worth 30 points each, and six on the opponents home side, worth 60 points each. The time limit was five minutes and a maximum wheel diameter of two inches was specified (an very unusual requirement for a contest). There was no limit on how many balls could be stored on the robot before depositing them in the scoring bin.

WHIZard's strategy was to cross the table, pick up as many balls on the opponent's side as possible, store them on the robot, and then return to deposit them in the scoring bin. In order to prevent the opposing robot from employing the same strategy, WHIZard was designed to be fast so that after picking up balls from the opposite end of the field, it could return in time to defend its home territory from slower opponents. Now one of the difficulties in crossing the ramps is for the robot not to get high ended, that is stuck on the chassis as the robot goes over the top of the ramp. The two inch wheel diameter limit is just enough to allow the 10 inch square WHIZard frame to clear the table top when taking the plunge over the lip of the ramp. To make sure the robot had enough traction and drive power, a four wheel drive design was used. In retrospect this was probably overkill but it had the benefit of moving the robot in a very straight path both forward and backward. Both motors on either side were given the same controller commands so that the robot handled much like a tracked vehicle, turning by means of what's called skid steering, rotating the wheels on one side in one direction, while the wheels on the other side rotate in the opposite direction. In practice this worked quite well.

Photo 4. The SoutheastCon 2001 Hardware Competition Playing Field (WHIZard inset).

Step 1. Motor Requirements.

The table is ten feet long and there are six, high scoring balls on the far side of the starting square. Although we would like the robot to pick all the balls up in a single foray, we should plan on making two passes in half the allotted 5 minute time limit. While the robot may be able to speed across the table, it's going to have to go quite slowly to locate and pick up the steel balls. If we make four passes across the four foot table *width* to pick up balls at a pokey two inches per second, that will take 96 seconds, or roughly 1.5 minutes per run. Difficulties involving robot-to-robot interactions are unpredictable and are left out of the present guesstimate. So then there remains one minute to traverse the ten foot length of the playing field four times. That gives WHIZard one minute to go forty feet, a speed of eight inches a second, not terribly fast. Since the torque for climbing the ramp will give a good acceleration on the flat sections, we can eliminate the acceleration distance in the calculation of motor speed, therefore:

$$\textbf{Motor Speed} = \mathbf{19.1} \times \mathbf{V_{avg} / D} = \mathbf{19.1 \times 8 / 2} = \mathbf{76\ RPM}$$

Since this playing field has a ramp, we need to find how much torque it takes to climb the ramp. As an initial guess, the WHIZard weight was estimated at 10 lbs, a good, average for robots in the eight to 10 inch size range (the final actual weight was 8 lbs). The torque just necessary to keep the robot on an incline, neither moving up nor rolling down, can be found from $T = 8 \times W \times D \times \sin(\theta)$. Plugging in WHIZard's estimated weight of 10 pounds (what was know at the time the robot was being designed), a wheel diameter of two inches, and a slope of 19 degrees,

Motor Torque = 8 x 10 x 2 x sin(19) = 52 oz-inches (all four motors acting together)

This is just the torque needed to keep the robot in place on the ramp, that is without rolling backwards. Since WHIZard will have accelerated on the flat part of the playing field, we don't have to add that to the hill climbing torque. However, we want more than just the minimum torque to keep the robot from rolling backwards so in our usual fashion let's double the torque to 104 oz-in. Dividing by four motors gives a torque requirement of 26 oz-in per motor. In summary our motor specifications (per motor)

Performance Requirement: **Torque = 26 oz-in, Speed = 76 RPM**

Step 2. Motor Specs. What motors did WHIZard actually use? Buehler model 61.46.032 gear heads with a no load speed of 400 RPM and a stall torque of 58 oz-in. Using these values and substituting a performance requirement of 76 RPM into the motor equations gives a motor torque of 47 in-oz. Thus each motor has an 80% reserve of torque. WHIZard's four motors have, at a speed of 76 RPM, a combined torque of 188 oz-in. Subtracting the 52 oz-in needed to maintain position on the ramp, there are an additional 136 oz-in available for maintaining speed or accelerating on the ramp. That's way more margin then necessary. This is a result of having four motors, which were chosen to insure smooth ramp transition and climbing. In summary for WHIZard and the Buehler motors:

$$\omega_o = 400 \text{ rpm}, \quad T_s = 58 \text{ oz-in}, \; Cr = 0.03, \; W = 8 \text{ lbs}, \; D = 2 \text{ inches}$$

Using the 26 oz-in torque requirement, each Buehler motor provides a speed of 220 RPM.

Step 3. Motor Operating Point. The torque operating point on the level part of the field is determined by the rolling friction. This gives a torque of just 3.8 oz-in on each wheel, which gives each motor a speed of 373 RPM. On the inclined portion of the playing field, each motor has an additional burden of 13 oz-in (on the uphill side) for a total of about 16 oz-in. The corresponding motor speed on the ramp is about 290 RPM, if the motors are run at full voltage.

Step 4. Cruising Speed. A motor speed of 373 RPM yields a cruising speed of 39 in/sec on the flats and 30 in/sec on the incline (for a conservative estimate we ignore the down ramp speed). At these speed, WHIZard would have traversed the ten foot long playing field in about 3.5 seconds. In practice, a time of 5 sec was typical, for an average speed of 24 inches/sec. Twenty four in/sec gives a new operating motor speed of 224 RPM.

Step 5. Acceleration Distance. Using the torque available for all four motors, stall torque of 58 oz-in each, and a cruising speed of 373 RPM on the level portion of the playing field, the acceleration distance is about S = 1 inch. Previously we made no assumption about acceleration distance.

Could WHIZard have used all the motor torque? Using the inclined plane method, the coefficient of static friction was measured as $\mathbf{C_S} = 0.73$. The max torque without slipping (for the actual weight) is: T = 8 x 0.73 x 8 lb x 2 in = 93 oz-in. Each motor only exerts a torque of 58 oz-in at most so there was no danger of slipping.

Why do we keep on over powering the robots? As you've noticed, coming up with motor requirements may, depending on the contest constraints and our experience, involve a lot of guess work. WHIZard had excess reserve, even more than we initially allowed for. However, the motors can always be operated at lower voltages if we desire to reduce the speed and torque. There are also other practical considerations. The motors chosen were small and fit on the platform, even considering the small wheel diameter limit, and were available at the time at a very good price on the surplus market. Our major goal in these motor specification determinations is to give us some guidance to choose among the myriad of motors available and to guarantee that our choice is more than minimally adequate, having plenty of reserve to allow for changes in platform design and operational strategy. If we choose under performing motors, we likely will have to redesign the basic mobility platform from scratch or suffer performance shortfalls. Of course, there are penalties for vastly over designing also, including possibly extra cost, weight, size, and power requirements.

How did WHIZard rank in the competition? Not very well overall, about in the middle of the pack. The problem was not the mobility but the navigation. For our present purpose of specifying motors for performance, WHIZard was number one – the only robot that successfully traversed the ramps. All the other robots stayed on the home court side of the playing field.

Example 3 – Head Banger: A Rail Mounted, Ball Returning Robot

Head Banger was built for the 2002 SoutheastCon IEEE hardware competition (once you start down this road, it's addictive). The contest was modeled on the early computer game of Pong. In the SoutheastCon contest, two robots volleyed a ball back and forth across a 4 foot by 8 foot court, each side of which was mildly inclined at 2.5 degrees (playing field shown in Photo 5). In order to keep track of the ball, each team was supplied with the output of a video camera mounted above the playing field. To score a point, a ball had to be deposited in a scoring bin behind each robot. The robots were confined to a 10 inch paddle zone across either end of the table. Up to ten plastic practice golf balls (like a wiffle ball) were dispensed from a central chute within the five minute time limit. The robot, no more than 8 inches wide, was allowed to travel along a structure mounted above and around the paddle zone. All competing teams chose a rail mounted robot of one kind or another to keep the robot within the confines of the paddle zone.

Head Banger was essentially a ball return carriage that was propelled back and forth across the end of the four foot wide table by a stationary, off board motor. The carriage assembly was attached to a pillow block, a self-aligning ball bearing sleeve, that slid along a precision ground rod. A drive motor and gear was mounted at one end of the rod, and an idler gear at the other end. A toothed timing belt ran around the gears and was attached to the carriage head. When powered, the motor turned, rotated the driving gear and the belt pulled the carriage head back and forth along the rod.

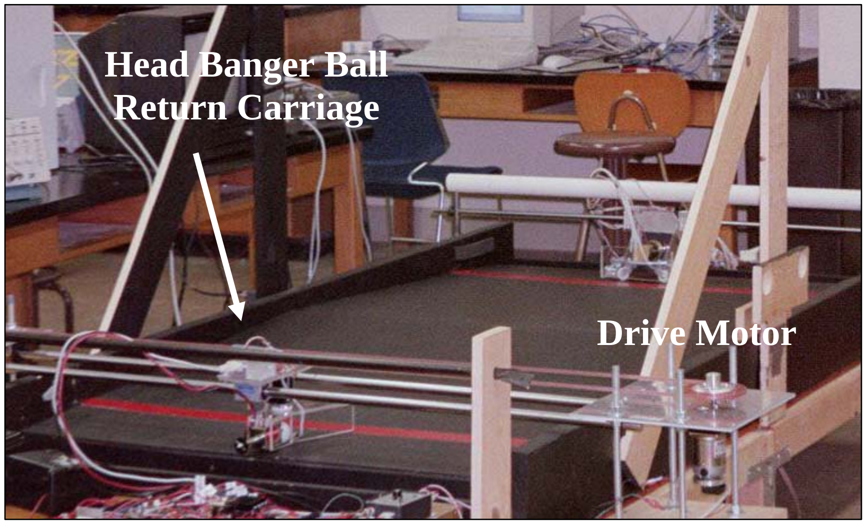

Photo 5. The SoutheastCon 2002 Hardware Competition Playing Field.

Step 1. Motor Requirements. The motor performance requirements were set by the table width, the transit time of the ball from one end of the field to the other, and the robot positioning control algorithm. The ball had a maximum return velocity above which it would jump or bounce out of the scoring bin and not score any points. By experimentally batting the ball back and forth and measuring the time with a stop watch, it was determined that the shortest transit time was about a half second. During that time the robot carriage head might have to travel the full width of the end zone to intercept an incoming ball, a distance of approximately 37 inches (a table width of 45 inches minus 8 inches for the width of the carriage head). Thus the *average* velocity required is about 74 inches/sec. Considering that carriage travel has to be controlled to prevent the robot from crashing into the support ends, that's really honking. During the early tests, the robot did indeed crash frequently, hence the name Head Banger.

What motor speed is needed to drive the carriage ball return mechanism at an average speed of 74 inches/sec? We are again confronted with choosing an acceleration distance **S**. As we saw in the Phoenix example, the motor speed is not sensitive to **S** when it is small compared to the range of motion. This time let's leave **S** as a variable and return to it when determining the torque, which is very sensitive to **S** values. We have an average velocity of 74 in/sec, a diameter of 4.5 inches (the driving pulley functions in the same way as a wheel for our analysis), a distance of 37 inches. Then,

$$\omega = 19.1 \times (1 + S/X) \times V_{avg} / D = 19.1 \times (1 + S/37) \times 74/4.5$$

simplifying and rounding a little,

$$\omega = 314 + 8.5\,S$$

The carriage head, including all moving parts, weighed 3.5 pounds. Using an initial estimate of 314 RPM, and substituting values,

$$T = \omega^2 \times D^3 \times W / 35{,}314 \times S = 890/S \text{ oz-in,}$$

a whopping number compared to our past examples. Small values of **S** are going to make this value even larger and yet not change the motor speed very much. Let's say we are willing to accelerate over a distance of six inches, and similarly decelerate over six inches on the other far end. Then, and we didn't discuss this before, we need to use 2*S, or 12 inches in the formula for ω but only S in the equation for torque. The reason is that when calculating average velocity we have to take into account both the accelerating and decelerating times. However it takes the same amount of torque to accelerate and decelerate. This actually makes the required torque more demanding because we have less space, hence time, to change the velocity. Therefore, using 2*S = 12 inches for the speed and S = 6 inches for the torque we have,

Performance Requirement: **ω = 416 rpm and T = 260 oz-in.**

Step 2. Motor Specs. The motor actually used, Japan Servo model DME60 with a 6H9F-H46 gear head, operated at 24 volts with a no-load speed of 550 RPM and a stall torque of 310 oz-in. Evaluating the motor equations with a speed of 416 RPM gives the Japan Servo motor a torque of only 76 oz-in. Looks like Head Banger would be grossly under powered. What to do? One thing we can do is change our strategy. Instead of racing from one end of the field to the other, we can park in the center and only have to go half the distance. After returning a ball, we again have time to return to the center before waiting for the return strike. The distance from the center to one end of the field is half as much as before, or 18.5 inches, for an average velocity of 37 in/sec. Now the speed and torque requirements are:

Performance Requirement: $\omega = 157 + 8.5\,S = 259$ **rpm,** $T = 100$ **oz-in.**

At a speed of 259 RPM, the servo motor has a torque of 164 oz-in. Head Banger's motor has a 50% torque surplus. The motor could also be operated at a higher voltage. Using 30 instead of 24 volts would provide approximately a 25% increase in torque and speed without seriously compromising the lifetime of the motor.

Could we have used a more powerful motor? For one thing, there isn't as much choice in the surplus market for this size motor as there is for smaller motors, and for another, the requirements aren't always known well in the beginning of the design process. For instance, in this case, we didn't have a good guess what the final return mechanism design nor what the overall weight of the carriage head would be. As a general remedy, I usually recommend purchasing two of any product one is not familiar with, if finances allow. Then one unit can be over tested and if a problem arises, or even if it doesn't there is a back-up. This can be especially valuable during contests, when components seem to fail far more frequent than one might imagine.

In summary, for Head Banger and the Japan Servo motor:

$$\omega_o = 550 \text{ rpm}, \quad T_s = 310 \text{ oz-in}, \quad C = 0.17, \quad W = 3.5 \text{ lbs}, \quad D = 4.5 \text{ inches}$$

Head Banger doesn't roll on wheels but slides on a steel rod. By tilting the rod until the carriage slowly moved along the rail under its own weight, an approximate value was obtained for the coefficient of sliding friction, given above as C = 0.17, almost six times the value we have been assuming for rolling friction.

Step 3. Motor Operating Point. Using the C = 0.17, the operating torque is 21 oz-in, which gives a motor speed of 513 RPM. This seems sufficiently higher than the 259 above that there should be no problem in whizzing across the rail. However, the required torque to accelerate Head Banger is 7 times more than the 21 oz-in to maintain a constant speed. The acceleration requirement is clearly going to dominate the motor performance.

Step 4. Cruising Speed. 513 RPM gives an cruising speed of 121 inches/sec. With an assumed acceleration distance of 6 inches, the average speed is 91 in/sec, better than our estimate of 37 in/sec.

Step 5. Acceleration Distance. The acceleration distance is 47 inches! What happened? While it may seem that we had done better by exceeding our required average speed of 37 in/sec, it takes longer and farther to achieve the higher speed of 91 in/sec. Long before Head Banger would accelerate to 91 in/sec, it would have traveled more than 37 inches and smacked into the end of the rail. On the other hand, the acceleration distance to reach a speed of 259 RPM, our requirement for an average speed of 37 in/sec, is (miraculously) 6.2 inches. When establishing requirements we chose 6 inches to see if we could find a value of S that would match the Head Banger motor to speed and torque requirements. Now we confirm that the given motor can achieve the goal of traveling half the length of the rail in 0.5 seconds.

What about our choice for S, how would our result have changed with a different value? Lower values of S will give larger torques, which may exceed the motor stall torque rating; higher values of S will give larger values of ω, which may exceed the motor no load speed. Looks like we made a lucky guess for S. Of course one can build a table with various S values to see if there is a matching point on a proposed motor speed-torque curve. When the acceleration distance is small compared to the length of the playing field, we can ignore it in the calculation of cruising speed and the torque to overcome friction or inclines dominates. When S is not small, for Head Banger S is 17% of the 37 inch travel distance, the torque to accelerate dominates. This was clear during testing of Head Banger. The speed of the return carriage increased visibly and the sound of the carriage sliding on the rail had a noticeable increase in pitch.

Motor Selection Summary

In brief, we've seen how to analyze contest rules to derive robot speed and acceleration requirements, how to calculate motor speed and torque from basic robot parameters like weight and wheel size, and we've applied these techniques to three examples drawn from real robots built for real contests. You may have noticed that the process is not necessarily straight forward but requires some serious thought and strategizing. There are also some techniques, like speed control, over volting, and wheel selection, that we can use to adapt available motors to our needs. Practical motor selection is one of the least understood techniques among hobby roboticists. We have presented a quantitative approach that takes the mystery out of the process, yet leaves you enough wiggle room to customize you're creations.

